Partnership work as a tool for professional development of mathematics teachers

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Abstract
The present text is part of a doctoral investigation in the narrative research modality, and it aims to identify and discuss collaborative work as an instrument of professional development through a partnership among teachers who teach mathematics. This investigation was carried out in a third-grade classroom of a public school in the state of São Paulo, where the researcher, who is a mathematics teacher, established a partnership with the pedagogue teacher in charge of the class. Data production occurred from March to July 2017, when the researcher and the partner teacher, under a collaborative perspective, planned, developed, recorded, evaluated, redesigned, and reflected on tasks focusing on spatial notions. The classes were videotaped, the task planning and analysis moments were audiorecorded and later transcribed and transformed into interactive episodes. Taking references from the historical-cultural perspective, Bakhtinian ideas, and narrative research, professional development is assumed to result from learning that is constructed in an alteritarian way, in the different interactions that the teacher establishes in the exercise of his/her profession. The text, based on episodes that comprise moments of interaction between the partner teacher and the researcher or between them and the children, provides evidence of how the intersocial and reflective processes of the partner teachers promoted learning and possible professional development.

Keywords: Spatial Notions, Collaborative Practices, Teacher Learning, Narrative Research.

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1. Introduction

For some years now, the subject of professional development has been at the forefront of research into the training of teachers who teach mathematics. Our aim with this text is to corroborate this discussion, taking professional development from a historical-cultural perspective. To this end, we have selected as the object of discussion a section of a doctoral research project carried out by the first author of this text and supervised by the second.

The research, with financial support from Capes, focused on the thesis: The writing of self through the weaving of narratives produced from the relationship with the other and through it as a practice of (self) formation and professional development (CUSTÓDIO, 2020). To carry it out, the researcher (who holds a degree in Mathematics) partnered up with a teacher-educator from the early years - referred to here as the partner teacher - due to their participation in collaborative groups, in which teachers have the opportunity to reflect collectively on pedagogical practices aimed at teaching mathematics at different stages of the educational process.

Using narrative research as a device to narrate the experience of (trans)formation and taking presuppositions from the historical-cultural perspective to discuss the role of the other in the processes of teaching, learning and development, the authors sought to identify and understand the instruments that can promote professional development. Narrative analysis modulated the weaving of pedagogical narratives - narratives produced in and from the practice of teaching mathematics - which intertwined different field texts, with the aim of interpreting and reflecting on the experiences of learning and teaching spatial notions in the early years of elementary school.

For this text, we sought to identify elements and practices of collaborative work, in partnership, which can become instruments for the professional development of teachers who teach mathematics. To achieve this, we have taken a section of this research, highlighting the movement of collaborative thinking established between the partner teacher and the researcher, present in the pedagogical narratives produced by the researcher, which make up the research text.

To better organize this text, in the first section we will outline our theoretical perspective, briefly presenting how collaborative work in a partnership can promote professional development. Next, we will present the methodological construction of the discussion: we will present how the data was produced and how it was analyzed. In the next section, we will discuss the interpretative-analytical approach. And finally, we will record some of our considerations.

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3 We will discuss our understanding of professional development later.
4 The research project was submitted and approved in 2017 by the USF Ethics Committee: Project No. 69595717.2.0000.5514.
5 This work was carried out with the support of the Coordination for the Improvement of Higher Education Personnel - Brazil (CAPES) - Funding Code 001.
6 Narrative research is made up of two types of text: field texts (in the case of this research, pedagogical narratives) and research texts, constructed through narrative analysis. However, when transforming narrative research into an article of a few pages, this movement between the two text modalities and the analytical one is not very evident.
2. Some theoretical assumptions about partnership work as a tool for professional development

To begin our discussion of how collaborative work resulting from a partnership can serve as a tool for professional development, we must first emphasize our concept of development.

From a Vygotskian perspective, human development is social and cultural, based on the "individual-social" dyad, in which we seek to understand how the social environment affects the development of psychological functions, and not the other way around, as is the case in other theories. The question is considered in terms of three relationships: between the social and the cultural; between the social and the symbolic; and between the social and psychological functions (PINO, 2005).

From the first relationship, everything that is cultural is social; on the other hand, the opposite is not true. In the second relationship, symbolic (sign) activity is the link between the biological and cultural levels - both in personal history and in the history of the human species. Symbolic activity acts on the dimension of consciousness because when a word is addressed to someone, it also mobilizes and produces changes in the speaker. On the other hand, but not in contradiction, from a Bakhtinian perspective, in sign activity, the sign is interpreted, assuming dynamism, mobility, and dependence on the enunciation context (BAKHTIN, 2011). The last relationship - between social and psychological functions - concerns higher psychological functions. From a Vygotskian perspective, these are inherently social, since "all forms of verbal communication between the adult and the child later become psychological functions. [...] All higher functions were formed in phylogeny, not biologically, but socially [...]" (VIGOTSKI, 2000, p. 26).

Therefore, we can state that the centrality of cultural development lies in the way social relations function in the interpersonal world, which is different from the way they function in the intrapersonal world, although they have the same nature (PINO, 2005). Thus, human development is linked to cultural development and can be understood as a social event, since both "the processes of mastering the external means of cultural development and thought" and "the processes of developing higher psychic functions" are primarily relations between people, i.e., intersubjective relations (VIGOTSKI, 1995, p.15 translation and emphasis added).

The appropriation of cultural forms of behavior involves the transformation of psychological activity and is based on the use of signs, of which the sign par excellence is the word. This appropriation involves the transformation of an initially external activity into an internal one, in a process in which interpersonal activity modifies intrapersonal functioning. These transformations are the result of a long development mediated by the other and by signs.

In this way, "human learning presupposes a specific social nature and a process by which children [or adults] penetrate the intellectual life of those around them". (VIGOTSKI, 2007, p.100).

But how can collaborative work be understood as a tool for professional development? To answer this question, we first need to understand the characteristics of collaborative work carried out in partnership.

Since the early 1990s, various researchers in mathematics education have contributed to the development of concepts related to collaborative
practices, collaborative work and collaborative groups (NACARATO; CUSTÓDIO; FRARE, 2021). In a historical and theoretical review, this work highlights that the book by the Research Working Group (GTI), Reflecting and investigating professional practice, published in 2002, became a reference for numerous discussions, especially the text by Boavida and Ponte (2002), in which collaboration is defined based on joint work, interaction, and sharing. The book takes up the work of Hargreaves (1998), Fiorentini (2004) and Passos et al. (2006). What is evident in these works is the role of collaborative practices in the professional development of teachers.

In addition, Nacarato, Custódio and Frare (2021) defend the contribution of Ibiapina’s (2008) work to deepen the discussion on the methodology of collaborative research, since this author, in her bibliographical review, links this approach to emancipatory practices and professional development from a Vygotskian and Bakhtinian perspective, our theoretical references. For her, collaborative research is a practice of continuous teacher training and takes place between peers who interact dialogically. The characteristics of collaboration are: reflective processes; a dialectical and dialogical movement between theory and practice; the production of meanings and resignifications about teaching practices.

We would also like to highlight the work of Robutti et al. (2016), in whose review the authors point out that few works theorize collaboration; in rare cases there are reports on how the theoretical maps adopted have shaped the research design; there are difficulties in relating teacher learning to collaboration within a project. In addition, although the authors identified many studies of teacher collaboration in the early and late elementary years, few highlight teachers from these two segments working and learning together. And when they do, the focus is not on understanding their processes to identify what they need to learn to teach.

For us, collaboration happens when the interactions that are established allow for contact with the different knowledge of each person involved in the process.

When these interactions take place and break down power relations, they aim at co-responsibility, respect, trust and allow the theoretical-methodological and experiential baggage from different professional and educational spaces to dialogue and contribute to processes of re-signification, be it of mathematical concepts, teaching practices and/or beliefs about education and mathematics teaching.

In this way, we understand that development in the professional sphere is also linked to penetrating the intellectual life of those around us, through interpersonal and intrapersonal relationships and reflections that, although they are distinct processes, form an inseparable pair. These relationships and reflections arise from interactions between teachers (in the case of this text, between the researcher and the partner teacher); objects of knowledge; students; official and unofficial documents; the school community; etc. We believe that "it is not what individuals are a priori that explains their ways of relating to others, but the social relationships in which they are involved that can explain their ways of being, acting, thinking and relating" (SMOLKA, 2000, p. 30).

Therefore, professional development is the result of these relationships, reflections and, consequently, of learning that is built in an alteritarian way, when
it comes from a partnership, since "[...] the knowledge conveyed by collaborative processes is formative in its very essence, since it creates zones of potential development [...]" (IBIAPINA, 2008, p. 50). Therefore, we can say that practices based on collaborative work, resulting from partnerships, can be understood as instruments of professional development.

In the next section, we will describe how the data was produced, how it will go through the analytical-interpretative process in this text and how it will help us understand partnership work as an instrument of professional development.

3. Methodological procedures

The data for the research took place from March to July 2017, with a 3rd grade class at a public school in the interior of the state of São Paulo. During this period, the researcher, who holds a degree in Mathematics, established a partnership with a teacher from this stage of education and, together, they analyzed textbooks for the 3rd year of elementary school, searching for how spatial notions were worked on in this age group; and in reading theoretical texts aimed at deepening these notions. In collaboration, the partner teacher and the researcher designed, planned, re-planned and developed the tasks for the classroom, aiming to develop elements that covered laterality, location and displacement, analysis and construction of location graphs.

All the lessons were videotaped, and the videos were analyzed together. The observations and conclusions identified served as support for continuing the work. The interpretations made through collaborative analysis of the videos or the records produced by the children were audio-recorded and then transcribed. The same happened with the video recordings, which were converted into transcribed texts and then into episodes. We understand that an episode should consist of a beginning, middle and end, with the duration of the moments of interaction being irrelevant. They were organized into turns (T) and numbered in ascending order (01, 02, 03...) to make it easier to identify the verbal interactions, which appear in italics. The explanations were written by the researcher and are enclosed in brackets and not italicized.

For this text, we will focus on the relationship established between the researcher and the partner teacher, to discuss the role of collaborative work - stemming from a partnership - as a tool for professional development. We will do this by taking a transcript excerpt from a moment of (re)planning tasks (episode), present in one of the pedagogical narratives produced by the researcher, in which spatial concepts were discussed. These clippings were considered by us to be field texts (CLANDININ; CONNELLY, 2015).

The analytical-interpretive procedures are based on the exploration of just one episode, also from one of the pedagogical narratives produced by the researcher, which shows the movement - carried out in collaboration - of redesigning a task proposed to the children in the classroom. This approach is in line with narrative analysis, in which the focus is on the interpretative perspective, which centers on the meanings produced by the actors in the investigation (BOLÍVAR, 2002).

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7 It is important to note that this partnership has existed since the author’s master’s research and that both the researcher and the partner teacher belong to the same research group led by her supervisor, the second author of the text.
To construct the narrative analysis, we intertwined the various field texts: transcripts of the planning session; episodes present in the pedagogical narratives; and excerpts from the researcher's reflective diary. This movement took place through the theoretical threads that helped in the reflection, interpretation, understanding and (re)signification of the experiences lived in the classroom.

In the next section, we will discuss, through the empirical data, how the partnership can be constructed as an instrument of professional development.

4. Partnership work as a tool for professional development

For this section, we will analyze the movement experienced by the partner teacher and the researcher when they planned and then proposed the task "Views of an object" to students in the 3rd year of elementary school in a public school in the interior of the state of São Paulo. We begin the discussion with the planning of the proposal, then we analyze the process in the classroom, when the teachers develop the task with the students, and we end the section with some summaries about working in partnership as an instrument of professional development.

4.1 Collaboratively planning the "Views of an object" task

When the work with spatial notions began in the classroom, the partner teacher and the researcher worked together on a sequence of tasks aimed at exploring these notions. It was known that the sequence would not be rigidly followed, as the movement of the children in the classroom would lead to reformulations. This is because, after each lesson, the researcher, and teacher would return to the records produced and, sometimes, to the video recordings, and this movement generated reformulations.

After starting to work in the classroom, the partner teacher identified that many of the tasks that had initially been planned would also appear as objects of knowledge in the geography curriculum. For this reason, she presented the researcher with the textbook adopted by the school (SIMIELLI, 2014), so that she could analyze the proposals and, together, think about reformulating the sequence of tasks planned at the beginning. This move ended up linking concepts that would be explored in geography classes with the work done in math classes. It became clear, then, that knowledge doesn't necessarily have to be fragmented; it can occur in a way that links the different fields of knowledge.

The dialog between the partner teacher and the researcher, presented in the following episode, highlights part of this process of reformulating the sequence of tasks:

Episode: Redesigning the sequence of tasks

T01 Teacher: So, did you like the book?
T02 Researcher: I liked it! I thought the tasks were really nice. And, as you said, there are things we've already done, right? And then, that one, the challenge, since you'd just started working on the views, and we'd already seen something in those tasks...
T03 Teacher: On page 18! [The book proposes cutting out different figures that represent the vertical, lateral, oblique and frontal views]. I didn't do 14, 15, 16 and 17. [Tasks involving working with these views using photographs].

T04 Researcher: Uh...

T05 Teacher: I'll do it this week.

T06 Researcher: Uh-huh...

T07 Teacher: So, 18, I thought the photography thing was cool.

T08 Researcher: Yes because that's what I thought: since we'd already worked on something there, you can work on it during the week, so you're not late8.

T09 Teacher: No problem!

T10 Researcher: Then, on Friday, if we could do it on Friday...

T11 Teacher: Yes, then!

T12 Researcher: On Friday, we'll do the photo because then we'll have time to get to the photo.

T13 Teacher: Cool!

T14 Researcher: Because when I saw that challenge, I thought: the school doesn't have the material, we'll have to take it.

T15 Teacher: No! They don't.

T16 Researcher: Then I thought: it's much cooler if we give them the camera... [When I saw the book's proposal, I immediately thought of providing the camera so that the students themselves could produce photographs with vertical, lateral, oblique and frontal views].

T17 Teacher: So they can go out around the school...

T18 Researcher: Yes!

T19 Teacher: From different angles, right?!

T20 Researcher: We give each group a camera, and they do the...

T21 Teacher: But what did you think? Did you think about, for example, taking a photo of the same thing from different angles or not? Three photos and then... Or is the challenge to photograph one thing, but from different angles?

T22 Researcher: So, that's what I was thinking and at the time I didn't decide, I said: "I'll talk to her..." 

T23 Teacher: Yes!

T24 Researcher: Because, first, when I thought about it, the first thing that came to mind was to propose that each group should do one. So, one group would do the front view, the other the top view and the other the oblique view, right?

T25 Teacher: Right!

T26 Researcher: But now you're talking, it's true, sometimes the same group can do all three views of an object...

T27 Teacher: Yes because then we can see if they understand what this view is...

T28 Researcher: Yes... Really!

T29 Teacher: Because then they have to dress up with their little bodies... And then we'll see if they understand.

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8 During one period of the field research, the researcher accompanied the classes on some days of the week, when the planned tasks were carried out with the class; on the other days, the teacher continued with her planning.
T30 Researcher: Yes. That's right... So they choose something and do all three.
T31 Teacher: The challenge is to choose something or someone and take it from all three angles.
T32 Researcher: Uh-huh... Yes!
T33 Teacher: The same thing!
T34 Researcher: Yes, that's right! It'll be cooler...
T35 Teacher: Because then they might come up with a lot of the same thing. Take three of the same pictures because they didn't understand the angle.
T36 Researcher: Yes... That's right!
T37 Teacher: Because you're working on angles there, aren't you? They're different angles, different views... the front view...
T38 Researcher: Yes... Aha... Actually, the question of views isn't directly about angles. You're not going to say it's an angle, but it's about positioning the camera, isn't it? The angle at which you place the camera is the view from which the photo will appear, isn't it?
T39 Teacher: Yes... it's the view that's going to appear!

The dialogue presented in the episode shows signs of the movement created between the partner teacher and the researcher. Although they already had a previously organized sequence, when the teacher noticed that the suggestions were close to the work that would be developed in geography, she wisely took the book to the researcher to compare the tasks and to think together about something that would allow them to connect the math and geography lessons. This move provides evidence that the partner teacher was concerned not only with her students' learning, but also with the researcher's findings and the learning that would be generated by the partnership.

When the researcher saw the book's suggestion to show the views (lateral, oblique, vertical, and frontal) through photographs, she immediately thought of giving the children (T16) a camera so that they could take the photographs themselves to represent the four views.

We believe that working in partnership, learning and thinking together is also a great contribution to the training of teachers and researchers. This movement of thinking together is made explicit in T21 when the partner teacher asked what the researcher thought of the suggestion that the students photograph an object, place, or person from the different perspectives: "But what did you think? For example, did you think about taking a photo of the same thing from different angles or not? Four photos and then... Or is the challenge to photograph one thing, but from different angles? This question raised two important issues. The first relates to the researcher, who sought to respect not only the teacher's classroom environment, but also her background as a teacher and researcher. At no time were decisions made in a solitary or alteritarian manner. Although the research was conducted by the researcher, the work was done in partnership; it was a co-construction.

The second factor that emerged from the teacher's interview is that the exchange between the researcher and the teacher proved to be very fruitful. They had different perspectives on the same proposal. The researcher, as a mathematics teacher new to research, had a more focused view of the concepts in her area of training. The partner teacher, an educator, researcher, and long-
time teacher, was very concerned about how these concepts would be developed with the children and how the proposals could generate more discussion and learning.

In this way, collaboration takes place because the interactions that are established allow for contact with the different knowledge of each person involved in the process. When these interactions take place and break down power relations, they aim at co-responsibility, respect, trust and allow the theoretical-methodological and experiential baggage from different professional and educational spaces to dialogue and contribute to processes of re-signification, be it of mathematical concepts, teaching practices and/or beliefs about education and the teaching of mathematics.

When she thought of proposing photography, the researcher's first idea was that each group would be responsible for recording one of the views of a given object, place, or person. However, the partner teacher sensibly identified that asking the same group to record all four forms of view of the same object, place, or person would be a strategy that would encourage more discussion. This stands out in T27, T29, and T35:

Yeah, because then we'll see if they're understanding what this vision is...; Because then they have to get ready with their little body... And then we'll see if they understand; Because then they might come up with a lot of the same things. Taking three identical photos because they don't understand the angle.

Finally, a question arose in T37: were we or were we not working with angles when we proposed tasks that explored the different views of the same object, place, or person? the partner teacher asked. And the researcher replied in T38: "Yeah... Aha... Actually, the question of views isn't directly about angles. You're not going to say that it's an angle, but to position the camera, right? The angle at which you place the camera is the view from which the photo will appear, right?".

We believe that it is only through relationships based on respect and trust that the other person feels safe to ask questions, expose their doubts and ideas. And this happened in this partnership relationship. The teacher felt safe in the presence of the researcher because she knew that her inclusion in the dynamics of her math class was not aimed at evaluating her work, but at analyzing the knowledge produced from and through the intersubjective and dialogical relationships established between the two of them and between them and the children. In this way, the production of knowledge took place much more intensely, as there was no fear of raising doubts and thinking about them together.

The dialog in this episode reveals signs of how productive the exchanges between the partner teacher and the researcher were. Having different backgrounds and working in different sectors, as well as having different careers, produced a range of issues to be discussed. Together, they helped each other; one learned from the other; and both from the children.
4.2 "Views of an object" the work in the classroom

With the students, the task began on June 9, 2017, when we organized the children into quartets and asked them to choose an object, place, or person to photograph. However, we were unable to produce and socialize the photographs on the same day. That's why the socialization was postponed until our next meeting on June 14.

As the school didn’t have the necessary equipment to carry out the task, we took cameras, tablets, and personal cell phones to the classroom and distributed them among the groups. They had to produce four photographs: one showing the object of choice from the front; another showing the side view; another showing the vertical view; and finally, one showing the oblique view of the same object.

We organized the groups and gave them the necessary instructions, explaining that each one had to choose an object, a person, or a space and produce four photographs that represented the frontal, vertical, lateral and oblique views. Before the children left to take their photographs, the partner teacher went over each of the types of views she had already worked on with them in geography lessons.

The children were very involved with the task, but we noticed that some of them had difficulty taking photographs using the lateral and oblique views. Many got confused with them; and in other cases, the object chosen was not conducive to photographic recording, as highlighted in the excerpt from the researcher’s reflective diary.

Figure 1 – Reflective research’s diary

Today, Wednesday, we started the lesson by socializing the photographs produced by the groups in the previous lesson. The students realized that some groups had not made good choices when it came to selecting the object or location to be photographed. Some groups chose the vegetable garden to be photographed, and it wasn’t possible to distinguish which photographs represented which view.

The bucket, for example, being cylindrical, made it difficult to differentiate between the side and front views, which were identical. One of the groups, which chose the garbage can as the object to be photographed, used the strategy of photographing the garbage can on which the label “garbage can” appeared, making it easier to identify the front view. The label did not appear on the side view.

However, the students who photographed the bucket didn’t have the same option. It was therefore difficult to differentiate between the photographs representing the front and side views.

Márcio’s group chose to record the bench. This made it easier to identify the photographs that represented each view.

(Reflective diary, June 14, 2017)

Source: Custódio (2020, p.188).

The questions raised in the excerpt from the researcher’s reflective diary came up when the photographs were socialized, as explained in the following episodes:

Episode 2: Socializing the photographs produced by Kauan’s group
[All the pictures presented below were photographs taken by the children]:

...
[...] Partner teacher: All right... A 2? [Refers to photograph 2, produced by Kauan's group].

**Figure 2 - Top view of the garbage can**

Original: Researcher's collection.

**Figure 3 - Top view of the garbage can**

Original: Researcher's collection.

T02 Vanessa: It's also from above.
T03 Partner teacher: Is it also from above? É? And do you agree, guys, that 2 is on top? [Some students agree, others don't].
T04 Rayssa: It's a bit sideways...
T05 Partner teacher: Is it a bit sideways?
T06 Rayssa: I think they've got the hang of it [makes a movement as if holding the tilted camera].
T07 Márcio: It's diagonal...
T08 Partner teacher: Is 2 diagonal, Márcio? Is it diagonal?
T09 Márcio: It's the same as Alisson's...
T10 Fernando: It's on top. It's up.
T11 Mikael: It's the same..., it's the same.
T12 Partner teacher: Got it! Okay. According to Márcio, he thought it was diagonal because of the way it is... It's different there... In number 1, it's closer, you realized that it's closer, right?! In the other one, they put the camera a bit further away, is that it?! He moved the camera away a bit [Márcio goes to the blackboard].
T13 Márcio: They took it here [shows the photo], but they took the front and the side.
T14 Partner teacher: They took a bit of the front and the side, that's why you thought diagonally? Different from the one over there?
T15 Márcio: Different from that one [shows figure 2].
T16 Partner teacher: Different from 1.
T17 Márcio: Because in figure 1, it only goes on top of the garbage. It doesn't get on top and a little on the side...
T18 Partner teacher: Right! Got it... What's up, Vanessa? You said it was vertical. Have you changed your mind now about what Márcio said or not? [Vanessa shakes her head in the affirmative]. [Vanessa agrees again]. Do you think we can convince you that it's the oblique view or not? [Vanessa agrees]. E? What about

T19 Partner teacher: What about four? [Figure 4]

**Figure 4 - Side view of the garbage can**

Source: Researcher’s collection.

T20 Rayssa: From the side...
T21 Márçio: From the front...
T22 Rayssa: From the side.
T23 Márçio: No! From the side...
T24 Fernando: Hey teacher! Vanessa’s talking...
T25 Partner teacher: Um... What about five?

**Figure 5 - Front view of the dumpster**

Source: Researcher’s collection.

T26 Márçio: On the side...
T27 Fernando: Teacher, Vanessa wants to talk...
T28 Some students: From the front...
T29 Partner teacher: [Teacher keeps thinking]. I think it’s the way the image is placed, isn’t it? Shouldn’t it be like this [vertically]?
T30 Researcher: No... I put it the way it was [referring to the way the photograph was taken].
T31 Partner teacher: Oh, that was...
T32 Researcher: Yeah... it’s the way it looks when they take it. I didn’t turn it so that wouldn’t happen!
T33 Partner teacher: Ah... But...
T34 Researcher: But if you turn it...
T35 Partner teacher: But if you turn it, I think...
T36 Researcher: If it rotates, it’s the same. I have them here...
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T37 Partner teacher: No! It's the same, but occasionally I get the impression that they confuse the side, they say it's sideways, but [moves hand showing vertical position], you know?
T38 Researcher: Oh, because of the position?
T39 Partner teacher: Because of the position...
T40 Researcher: There they are, you want them to open...
T41 Teacher partner: No! You don't have to... But you see? I don't know... It could be my doubt.
T42 Researcher: Uh... But it could be...

The episode begins with an analysis of the second photograph produced by Kauan's group. According to Vanessa, one of the group members, the second photograph, like the first, also represented the view from above the garbage can. This statement caused disagreement among the children, as some of them agreed with her, but the rest did not. In T04, Rayssa said: "It's kind of sideways...". And she continued in T06, arguing: "I think they've got the hang of it [makes a movement as if holding the tilted camera]". Márcio agreed with his colleague and added, in T07: "It's diagonal...".

The teacher began to question, allowing Márcio to argue his hypothesis and convincing Vanessa and the rest of her classmates who agreed with her statement that the photograph did not represent the vertical view, but the oblique view. And, on T13 and T17, he said: "They took it here [shows the photo], but they took the front and the side"; "Because 1, it only takes in the top of the garbage. She doesn't pick up the top and a bit of the side...". In this process, the hypotheses elaborated are put up for discussion in the interactive context created by the dynamics of collective socialization, and intersubjective relationships are established, enabling new meanings to be given.

Márcio's arguments in turns 13 and 17 reveal signs of the development of the concept of oblique view. He was able to clearly describe the characteristics of a photograph that represents the oblique view: the side and part of the upper side of the object appear in the photograph. With his arguments, he convinced Vanessa that the second photograph was a representation of the oblique view, as indicated in T18.

The dialogue continued, and a new question was raised by the teacher when the fifth photograph was analyzed. Some children said that it (Figure 6) represented the front view and others considered the side view, even though the garbage can label was visible.

Based on Vygotskian studies, we understand that conceptual development is described by a long path, "[..] and it is through this path that the subject, when learning a new word, becomes aware of impressions that give him the possibility of constructing the meaning […]. According to the subject's development, the word acquires a certain meaning, and this will be expanded over the course of the process, depending on the progress of the path to be traveled, remembering that this path is not linear." (CUSTÓDIO, 2016, p.76). For Vygotsky (2009), the functional use of words is the crucial point in the development of concepts. With it, children and adults become capable of controlling their psychological processes and directing them towards solving problems.

The students’ disagreement led to a reflection by the teacher, who, in T29, asked me: “I think it's the way the image is placed, isn't it? Shouldn't it be like this [vertically]?” The researcher didn't understand the teacher's question and said, in T30, that she hadn't changed the position of the photograph, but had inserted it into the slide show in the way it had been produced.

The dialog between researcher and partner teacher continued, and the researcher stated, at T36, that even if she rotated the image, it would remain the same. Until then, she hadn't understood that it was the fact that it was horizontal (landscape orientation) that could have led the children to think about the side view. And the teacher tried to explain again in T37: "No! It’s the same, but sometimes I get the impression that they confuse the side, they say it's sideways, but [moves hand showing vertical position], you know?” Only then did the researcher understand what the teacher's hypothesis was. There is evidence of this understanding when she asked, in T38: "Oh, because of the position?”. And she agreed.

This move once again highlights the importance of the partnership and the way in which it led to a lot of reflection and learning. Based on the diversity of hypotheses put forward by the children, the teacher sensibly identified that the problem causing the confusion was the position of the photo. There is evidence that the children defended the hypothesis that the image represented the side view, due to the position of the photograph (the image was in landscape position and not portrait). Through the established dialogic movement, the researcher listened in an alteritarian way to what the more experienced partner teacher had to say, and together they managed to understand and subsequently solve the problem.

The discussions continued. Then the question of the shape of the garbage can came up, as highlighted in Episode 3:

**Episode 3:** "[...] the garbage can is round [...] you can't take the two different positions"

_T01 Partner teacher: Speak up, Vanessa!_  
_T02 Vanessa: When I went to take the four, I didn't know which was the front of the garbage can..._  
_T03 Partner teacher: Oh, that's great! Guys, look what Vanessa's saying: when she went to take out the four, she didn't know which_
side was the front of the garbage can. Why does that happen, Vanessa?

T04 Márcio: Because the garbage can is round, not square, not rectangle... [Alisson turns his finger in a circular motion].

T05 Ana Paula: Because the garbage is round.

T06 Fernando: But it does...

T07 Rayssa: Because you can't take the two different positions.

T08 Partner teacher: Okay!

T09 Rayssa: There's no tip!

T10 Partner teacher: Very nice, that's it! You can see how interesting it is, then you can't!

T11 Nicolly: Hey, buddy! I realized that four is on the side and five is on the front because here four, oh [goes to the blackboard], doesn't have the same paper [refers to the garbage can label].

T12 Partner teacher: Oh, right! So, you think it's from the front because there's the little paper?!

T13 Fernando: That's what I was going to say, Dad! There's something that shows it's from the front...

T14 Kauan: We wanted to show that here [shows photo 4] it was from the back and here [shows photo 5] it was from the front.

T15 Partner teacher: Because of the paper?

T16 Kauan: Yes! I wanted it because of this [shows the paper].

T17 Partner teacher: Ah, so the paper was the point of reference for you. If you hadn't had the paper, how would you have known it was the front?

T18 Kauan: I would know that it's the same... It was going to be right here. I would pass a little piece [referring to demarcating the front], and then it would be from the front. If it wasn't the same as what I was going to do here, I was going to take it from the back.

T19 Márcio: But then it would be the same photo. It would be the same.

T20 Partner teacher: It would?

T21 Nicolly: It would be the same.

T22 Partner teacher: It was going to be the same. Because that's what they're talking about [rotates fingers in a circular direction], it's the shape of the garbage that complicates it for us... We don't know where it is from the side or the front...

In Episode 2, Fernando warned his partner teacher several times that Vanessa would like to speak. However, the discussions continued, and it was only in Episode 3 that Vanessa managed to say what she wanted, as indicated by T02: "When I went to take the four, I didn't know which was the front of the garbage...". However, her group used the strategy of differentiating the photograph from the front view by taking the position where there was a label on the garbage can as an indication that that was the front of it.

Vanessa's speech prompted the teacher to problematize: "Oh, that's great! Guys, look what Vanessa is saying: when she went to take out the four, she didn't know which side was the front of the garbage can. Why does that happen, Vanessa?". The movement of alteritarian listening (SOBRAL; GIACOMELLI, 2020) presupposes a response that is also alteritarian, and this response occurs through problematization.

The children also got involved in the movement of alteritarian listening and, immediately after the problematization was launched, several responses
Episode 4: "[...] I think they turned the camera wrong"

[...]
T01 Researcher: What did you want... Rayssa wanted to talk...
T02 Partner teacher: Go ahead, Rayssa.
T03 Rayssa: No! It's just that there, they took it from the front, but I think they turned the camera the wrong way [moves his hands as if he were holding a camera]. They took it sideways [refers to positioning the camera sideways].
T04 Partner teacher: So that's why! Understand?
T05 Researcher: Hum...
T06 Partner teacher: You think you're lying down? But actually, it's the way... they take it right, from the front, like this [shows the front position]. It's just that the photo, when you look at it like that... may I?
T07 Researcher: Yes!
T08 Partner teacher: Oh, it's like an image... I didn't know that!
T09 Researcher: Yes! I'll go back... I put it there so it wouldn't be too busy...
T10 Partner teacher: Just to clear up their doubts because when we look at it, it seems like... That's the one, see? Turn it...
T11 Researcher: Uh-huh...
T12 Partner teacher: See?
T13 Márcio: It's from the front!
T14 Partner teacher: Is that what you were wondering? It's because the image isn't rotated, that's why!

In T03, Rayssa returned to the question raised by her partner teacher in Episode 2 when she said: "No! It's just that, there, they took it from the front, but I think they turned the camera the wrong way [moves her hands as if she were holding a camera]. They took it from the side [refers to positioning the camera sideways]". Rayssa agreed with her colleagues' arguments that the fifth photograph represented the front view, but, for her, the garbage can be lying down in the image led to the conclusion that "[...] they turned the camera wrong [...] They take it from the side". The teacher resolved the doubt by rotating the image and putting it in portrait position. And, once again, the alteritarian listening
movement made it possible to identify and understand what was causing the student's doubt.

4.3 Views of an object: partnership as a tool for professional development

The episodes presented provide evidence of the movement of reflection that took place between the researcher and the partner teacher, and between them and the students in the classroom.

Although interpersonal and intrapersonal reflection represent distinct moments, they form indivisible units in the process of professional development and the formation of reflective awareness, since one level helps to build the other. In this sense, they are not disparate processes; on the contrary, they form an inseparable unit (IBIAPINA, 2008, p. 46-47).

Ibiapina's (2008) contributions to collaborative work touch on the discussion we have been having about the potential of partnerships as a tool for professional development. The author uses Vygotskian and Bakhtinian concepts to discuss collaborative work, arguing that interpersonal and intrapersonal reflective movements act as indivisible units in professional development and in the constitution of a "reflective conscience". We believe that this also occurs in partnership work.

When, together, the partner teacher and the researcher studied the concepts related to spatial notions, (re)planned tasks and developed them in the classroom; or when, after the movement experienced with the children, they returned to the records and video recordings, seeking hypotheses for the dynamics that had arisen, they were engaged in interpersonal and intrapersonal movements of reflection. They were interpersonal because they were thinking together and based on the children's elaborations and each other's constructions. They were also intrapersonal because they mobilized knowledge from different training spaces and moments for each of them. The articulation between both facets of reflection led to the formation of a reflective consciousness.

We believe that the movement created by the researcher and partner teacher mobilized "zones of potential development", which in turn led to the expansion of concepts through processes of (re)signification. This reconversion depends on more capable partners and sign mediation, in which the word takes on the role of an ideological sign (VIGOTSKI, 2009; VOLÔCHINOV, 2017).

In partnership work, there is no dependence on more capable partners, but rather on partners with experiences and knowledge from different educational and professional contexts and spaces. In this way, the mobilization of concepts already internalized by both parties, in other words, the retaking of the conceptual prehistory of the partners involved, facilitates the learning process and leads to development in a dialectical, dialogical and alteritarian way.

However, such processes are only possible through enunciation, because only in this way does think become accessible to intersubjective relations (VIGOTSKI, 2009). From a Bakhtinian perspective, thought is alteritarian and represents the ways in which discourses affect and allow themselves to be affected in dialogical relationships. We therefore see partnership work as a potential space for collaborative movement.
5. Some considerations

In this text, we aim to identify and discuss collaborative work as an instrument of professional development. To accomplish this, we present excerpts from moments of data production, taken from pedagogical narratives produced by the researcher, which show moments of working in partnership, highlighting the dialogic and alteritarian movement between the researcher and the partner teacher and between them and the students in the classroom. In presenting the episodes with the students, we aimed to show how the classroom is marked by unpredictability and to point out that, even though the teacher and the researcher had planned the task, the paths taken by the students were different from those planned. However, the dialogic movement that took place in the classroom allowed the students to be heard and their ideas and arguments to be analyzed and debated collectively. In this investigative stance of the teacher and the researcher, knowledge was elaborated, re-elaborated, given meaning and re-signified.

These excerpts show evidence of how partnership and collaboration, made explicit: in the movement of thinking together about a task to be developed in the classroom; carrying out its re-planning, after collaborative studies and reflections; reflecting on the students' productions and, based on them, mobilizing their knowledge; developing alteritarian listening and speaking, which allow and subsidize the construction and continuity of a dialogue woven without power relations that lead to silencing; are configured as instruments of professional development. These tools can promote interpersonal and intrapersonal reflections, creating zones of potential development and enabling learning.

We reiterate that teachers learn in and from the context they live in and the social relationships they establish, which, as Smolka (2000) states, explain their ways of being and acting. It is important to note that both the researcher and the partner teacher had no experience of teaching spatial notions. The research enabled both of them to learn, such as expanding concepts relating to spatial notions; the importance of collaboration, from the study of a theme or field of knowledge to the development of tasks, led to discussions that created contexts of meaning or re-signification of concepts. Specifically for the researcher, a mathematics teacher, the lessons learned include understanding how to work on concepts related to this field of knowledge in an interdisciplinary way; how to problematize and mediate based on listening and speaking in other ways; how to take advantage of a student's speech or hypothesis as the generator or guiding thread of a discussion.

In this way, we understand that development in the professional sphere is also linked to penetrating the intellectual life of those around us, and this occurs through interpersonal and intrapersonal relationships and reflections which, although they are distinct processes, form an inseparable pair. These relationships and reflections come from the interactions between teachers (in the case of this text, between researcher and partner teacher); objects of knowledge; students; official and unofficial documents; the school community; etc. Therefore, practices based on collaborative work, stemming from partnerships, can be understood as instruments of professional development.
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